WESTMINSTER SCHOOL
THE CHALLENGE 2019

MATHEMATICS III

Wednesday 1 May 2019

Time allowed: 1 hour 30 minutes

You may not use a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

1   *Bestco* sells strawberry yoghurt in 180 gram pots for 78 pence each.
      Gainsbury's sells strawberry yoghurt in 155 gram pots for 65 pence each.
      Which supermarket offers better value for money? Show clearly how you decide.

2   **a**   Two numbers are in the ratio 3 : 5 and have a lowest common multiple of 90.
         What are the two numbers?
    b   Three numbers are in the ratio 10 : 12 : 15 and have a lowest common multiple of 300.
         What are the three numbers?

3   The diagram shows a path round the perimeter of a 400 m by 100 m rectangle.

Pagoda	400 m	**Obelisk**
100 m		
Sundial		**Fountain**
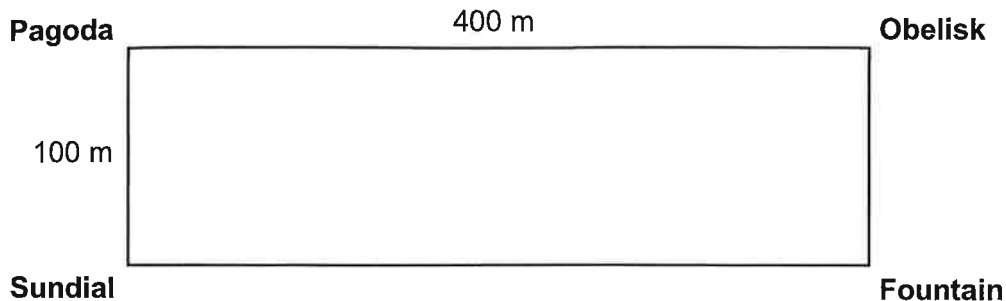
Calum and Xiaoming race from the pagoda to the fountain. Calum runs at 8 metres per second to the obelisk, and then at 5 metres per second from the obelisk to the fountain.

Xiaoming starts at the same time as Calum does, running at 6 metres per second to the sundial. At what speed does he need to run from the sundial to the fountain in order to reach the fountain at the same time as Calum?

4   A school is divided into a Junior section and a Senior section. The Head Teacher does an inspection of school uniform and finds that 65% of the Juniors but only 25% of the Seniors meet the uniform standards.

Given that only 43% of the whole school meet the uniform standards, what percentage of the school are Juniors?

5   In the diagram below, PQR, PQS and PQT are all corners of regular polygons.
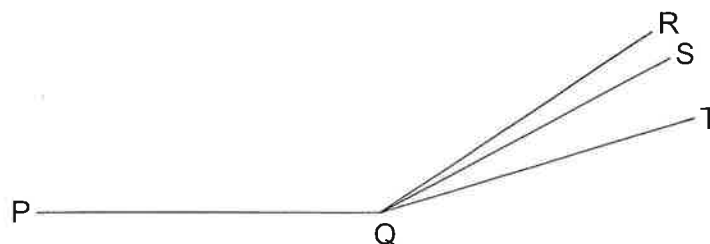
- The middle polygon has 18 sides.
- The largest polygon has twice the number of sides of the smallest polygon.

Given that angle SQT is twice the size of angle RQS, determine the number of sides of the smallest and largest of the regular polygons.

6 **a** Some pupils from Rugby and Sedbergh come together for a joint school trip. The caterer makes up two separate bags of sandwiches, one for each of the two schools, so that each pupil should get one sandwich each.

Unfortunately, the labels bearing the names of the schools become switched, so that at lunchtime, each Rugby pupil receives an extra two fifths of a sandwich.

What fraction of a sandwich does each Sedbergh pupil get to eat?

b Some pupils from Westminster, Eton and Harrow come together for a joint school trip. The caterer makes up three separate bags of sandwiches, one for each of the three schools, so that each pupil should get one sandwich each.

Unfortunately, the labels bearing the names of the schools become switched, so that at lunchtime, each Harrow pupil receives an extra three eighths of a sandwich and each Eton pupil only receives four fifths of a sandwich.

What fraction of a sandwich does each Westminster pupil get to eat?

7 **a** Write an equivalent expression to
$$a - \left(b - \left(a - \left(b - a - b \right) - a - b \right) - a - b \right)$$
without using brackets.

b In each case insert brackets into the expression
$$a - b - a - b - a - b - a - b - a - b$$
to give an expression which simplifies to
  i $3b - a$
  ii $5a - 3b$

c In each case explain why no combination of brackets inserted into
$$a - b - a - b - a - b - a - b - a - b$$
could simplify to
  i an expression containing b only,
  ii $\left(\text{a whole number multiple of } a \right) + 5b$.

8 In each part of this question two cyclists race each other. Determine which rider wins the race, and how many seconds later the other rider finishes. You may assume each rider cycles at constant speed.

a Archibald is 3 km from the finish and Boardman is 500 m behind.
45 seconds ago, Archibald was at Boardman's current position.
In 40 seconds' time, Boardman will be in Archibald's current position.

b Cavendish is 9 km from the finish line.
6 minutes ago, Cavendish was in Darke's current position.
In 5 minutes time, Darke will be in Cavendish's current position.
Darke is cycling 3 kilometres per hour faster than Cavendish.

9 In the diagram below, $AB = AD$ and $CE = CD$. Let angle $ABC = x$.
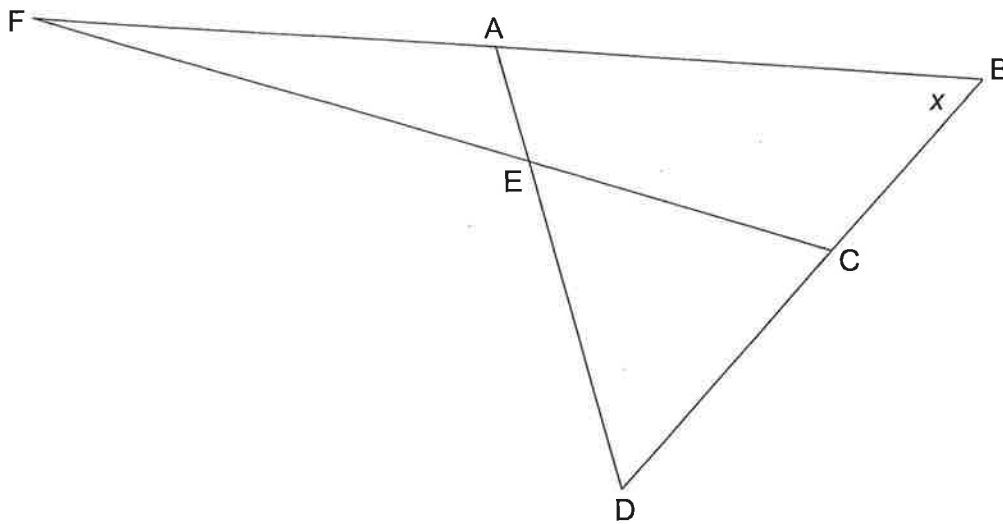
a Find an expression, in terms of x, for angle ECB.

b Show that angle ABD must be less than $60°$.

10 The diagram shows two squares, one inside the other. The area of the smaller square is 490 cm^2. Distance OB is three times distance OA. What is the area of the larger square?
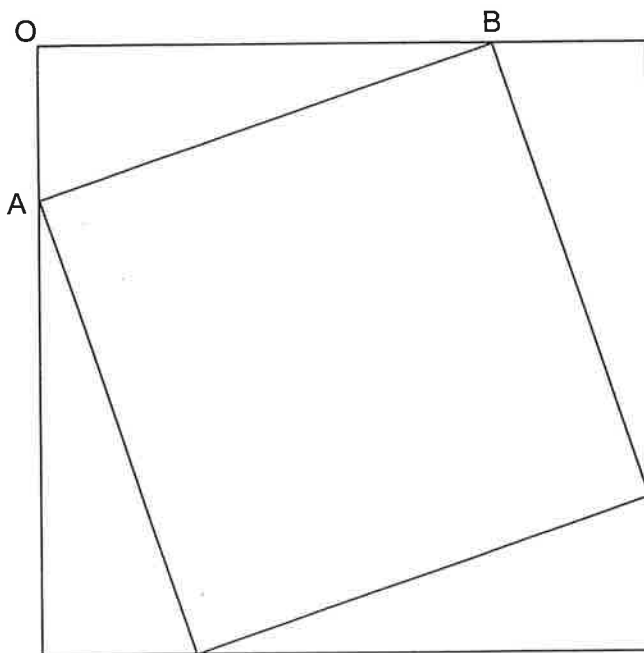
11 In this question you may assume that a positive whole number is a multiple of 3 if the sum of its digits is a multiple of 3.

    a By writing 1111 as $1110 + 1$ and 11111 as $11100 + 11$, show that 1111 is 1 more than a multiple of 3 and 11111 is 2 more than a multiple of 3.

    List **A** is defined as follows:
- The list starts with 1.
- If T is a number in the list, $10T + 1$ is the next number in the list.

    b Write down the eighth and ninth numbers on the list. What are their remainders when divided by 3?

    Lists **B** and **C** are defined as follows:
- Each number on list **B** is equal to one more than five times the corresponding number on list **A**.
- Each number on list **C** is equal to the corresponding number on list **A** plus the corresponding number on list **B**.

    c **i** Explain why the 1st, 4th, 7th, 11th (and so on) numbers on list **B** must be multiples of 6.
       ii Explain why every number on list **C** has the same remainder when divided by 3. What is that remainder?

12 **a** Find the prime factorisation of 108.
    b List all the factors of 108 (including 1 and 108), and find their sum.

    The following diagram can be used to illustrate another way of finding the sum of the factors of 108.


    c Use this diagram to explain why the sum of the factors of 108 is 7×40, and hence verify your answer to part **b**.
    d *Use a similar method* to find the sum of the factors of 400.
    e A pupil who has misunderstood this idea writes 4500 as $5^3 \times 6^2$ and deduces that the sum of the factors of 4500 is $43 \times 156 = 6708$. Is this answer smaller or larger than the correct one? Explain your answer.
    f The number 135169 can be written as $29 \times 59 \times 79$. Find the sum of the factors of 135169.

13 A smartphone is unlocked when the user traces out the correct sequence of dots on a lock screen. The dots are arranged in a 2 by 3 grid.

The sequence must satisfy the following rules:

- The sequence must use exactly three dots;
- The dots must be joined by a continuous path made up of only vertical, horizontal, and 45° diagonal segments;
- The same dot cannot be used twice in the sequence.

One valid sequence is shown in the diagram.
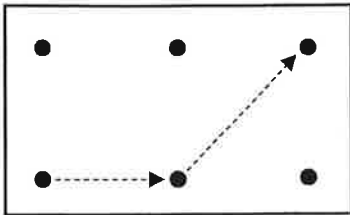
a How many valid sequences start at the upper-left dot?
b How many valid sequences start at the upper-centre dot?
c Use your answer to parts **a** and **b** to show that there are 64 valid unlock sequences in total.

Poppy and Harriet both have this kind of smartphone. Unfortunately, Poppy has smashed her screen, so she cannot use any sequence that involves the upper-left dot. Harriet has also smashed her screen, so she cannot use any sequence that involves the upper-right dot.

d Find the number of valid sequences that *neither* girl can use.

Poppy and Harriet want to read all of each other's messages, so they agree to use the same unlock sequence on both of their phones. You may find it helpful to consider a Venn diagram for the remaining questions.

e How many valid sequences can they choose from?
f How many valid sequences are usable on Harriet's phone?

14 The diagram below shows a rectangular piece of paper ABCD, measuring 4 cm by 10 cm, before and after it has been folded so that corners A and C meet.
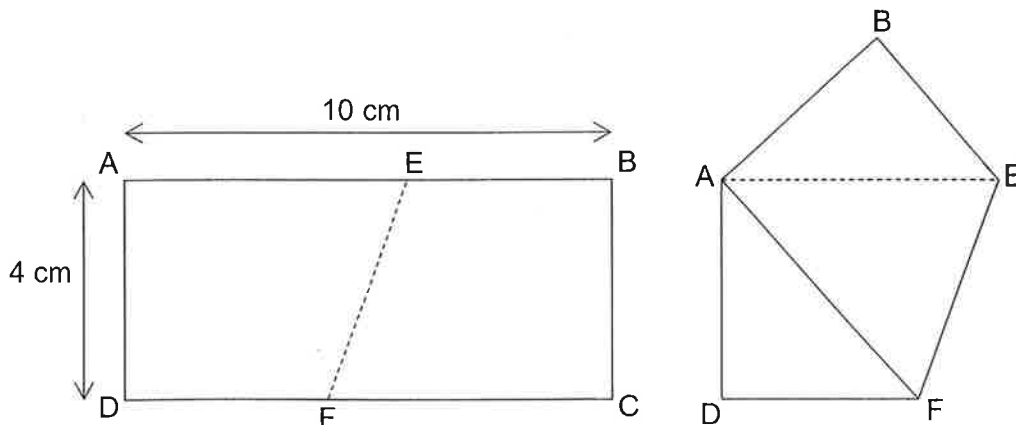
Calculate the length AE, showing detailed reasoning and working.